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Abstract

We propose a tractable, model-based stress-testing framework where the solvency risks, funding liquidity risks and market risks of banks are intertwined. We highlight how coordination failure between a bank's creditors and adverse selection in the secondary market for the bank's assets interact, leading to a vicious cycle that can drive otherwise solvent banks to illiquidity. Investors' pessimism over the quality of a bank's assets reduces the bank's recourse to liquidity, which exacerbates the incidence of runs by creditors. This, in turn, makes investors more pessimistic, driving down other banks' recourse to liquidity. We illustrate these dynamics in a calibrated stress-testing exercise.

JEL classification: G01, G21, G28, C72, E58
Bank classification: Financial stability; Financial system regulation and policies

Résumé

Les auteurs proposent un cadre d'analyse maniable pour la réalisation de tests de résistance, fondé sur un modèle dans lequel les risques de solvabilité, les risques de liquidité de financement et les risques de marché des banques sont interreliés. Ils montrent de quelle manière l'interaction entre la non-coordination des créanciers d'une banque et l'antisélection de ses actifs sur le marché secondaire mène à la formation d'un cercle vicieux pouvant conduire des banques autrement solvables à une situation d'illiquidité. Le pessimisme des investisseurs à l'égard de la qualité des actifs d'une banque limite les possibilités dont elle dispose pour avoir accès à des liquidités, ce qui exacerbe l'effet des désengagements massifs par les créanciers. S'ensuit un renforcement du pessimisme des investisseurs qui restreint les possibilités de recours aux liquidités d'autres banques. Les auteurs illustrent cette dynamique à l'aide d'un test de résistance calibré.

Classification JEL : G01, G21, G28, C72, E58
Classification de la Banque : Stabilité financière; Réglementation et politiques relatives au système financier

Non-Technical Summary

Stress-testing is an invaluable tool to identify vulnerabilities within financial systems. Alternatively, stress tests can be repurposed to support crisis management and resolution. A criticism of many stress-testing models, however, is that they are largely partial equilibrium and do not feature adverse feedback effects between market risk and funding liquidity risk, which were at the heart of the recent global financial crisis.

In this paper, we present a new, model-based stress-testing framework where the solvency risks, funding liquidity risks and market risks of banks are intertwined. We highlight how coordination failure between a bank's creditors and adverse selection in the secondary market for the bank's assets interact, leading to a vicious cycle that can drive otherwise solvent banks to illiquidity. Investors' pessimism over the quality of a bank's assets hampers the bank's recourse to liquidity, which influences the incidence of bank runs. This, in turn, makes investors more pessimistic, driving down other banks' recourse to liquidity even further.

Under the global games framework, bank runs are endogenous in our dynamic stress-testing model. A run on a bank is driven by the bank's credit and market losses, its funding composition and maturity profile, and concerns that creditors may have over its future solvency. In each iteration of the model, creditors of solvent banks are given the opportunity to withdraw and continue lending. Investors update their belief about the quality of the assets at the end of each iteration.

We demonstrate these dynamics in the context of a calibrated stress-testing exercise inspired by Canada's 2013 Financial Sector Assessment Program (FSAP), which was administered by the International Monetary Fund (IMF).

1 Introduction

The global financial crisis was put in motion by the announcement of the French bank BNP Paribas on August 9, 2007, that it had suspended withdrawals from three of its investment funds that were exposed to U.S. subprime mortgages. This news triggered anxiety among market participants regarding other banks' exposures to subprime mortgages, which was exacerbated by the opacity of banks' balance sheets. As the crisis persisted, funding conditions deteriorated for all banks, culminating in the worst economic downturn since the Great Depression.

Policies to shore up confidence in banks and prevent future crises have been developed along several dimensions.[1] Concurrently, stress testing has emerged as an invaluable tool for assessing risks to the banking system. Such *macro* stress tests, according to Borio et al. (2012), may be used to identify vulnerabilities within financial systems. Alternatively, they can also be repurposed to support crisis management and resolution. A criticism of many macro stress-testing models, however, is that they are largely partial equilibrium and do not feature adverse feedback effects between market risk and funding liquidity risk, which were at the heart of the recent crisis.

In this paper we present a new, model-based stress-testing framework where banks' solvency risks, funding liquidity risks and market risks are intertwined. We highlight how coordination failure between a bank's creditors and adverse selection relating to the value of the bank's assets interact, leading to a vicious cycle that can drive otherwise solvent banks to illiquidity. Investors' pessimism over the quality of a

[1]For solvency risk, banks are required to hold more, and better quality capital to guard against adverse credit shocks. For liquidity risk, minimum liquid asset buffers and a matching of asset and liability maturities across balance sheets are codified under the Liquidity Coverage Ratio (LCR) and Net Stable Funding Ratio requirements for banks. These rules are at the core of the Basel III regulatory standard, which was negotiated and agreed by the Basel Committee on Banking Supervision in the aftermath of the crisis.

bank's assets hampers the bank's recourse to liquidity, which influences the incidence of bank runs. This, in turn, makes investors more pessimistic, driving down other banks' recourse to liquidity.

A bank's funding liquidity risk is endogenous in our stress-testing framework. The risk is driven by the bank's credit and market losses, its funding composition and maturity profile, and concerns that creditors may have over its future solvency. Using the global games framework of Morris and Shin (2003), we derive a unique solvency threshold for each bank: whenever losses are greater than a certain threshold, creditors run, and the bank fails. Market risk is also endogenous in our model and depends on the pervasiveness of adverse selection, which is driven by two components: the *quality* of banks' assets and the prior beliefs that investors vying to buy assets have about the quality of the assets. Pessimistic investors are more likely than optimistic investors to believe that assets are of low quality.

Investors offer banks a pooling price for their assets, due to asymmetric information, and the more pessimistic the investors, the lower the pooling price. This, in turn, reduces a bank's recourse to liquidity and its ability to service withdrawals and maturing liabilities. As banks begin to fail due to funding liquidity risk, investors Bayesian update their beliefs about the quality of the assets. If it is more likely that a bank will fail because its assets are of low quality than that the bank will survive because its assets are of high quality, investors become more pessimistic. This can lead to a vicious cycle of information contagion, and illiquidity spirals between investors' pessimism and banks' funding liquidity risks emerge.

The model also exhibits virtuous information contagion and liquidity cycles. Suppose that investors are reasonably optimistic to start with, and that most banks survive following credit shocks. If it is more likely for a bank to survive because its

assets are high quality than for the bank to fail because its assets are low quality, then investors become more optimistic. This, in turn, improves other banks' recourse to liquidity and thereby mitigates their funding liquidity risk. Indeed, the public disclosure of the results of the Supervisory Capital Assessment Program in 2009 by banking supervisors in the U.S. credibly demonstrated the resilience of banks and restored confidence in the banking system (Bernanke, 2013).

The influence of changes to the prices of good- and bad-quality assets may be mapped to the price spread. An increase in the spread leads to an increase in the vicious illiquidity channel. A larger spread is associated with greater uncertainty in the quality of the assets. When investors update their beliefs, they are more inclined to assume that the likelihood that some banks failed because their assets were low quality is greater than the likelihood that other banks survived because their assets were high quality.

With a view towards stress testing, we theoretically demonstrate that our iterative model set-up converges to a unique solution after a finite number of iterations. Using a simple inductive argument, we show that in a stress-testing environment involving $N \geq 2$ banks, the cycles of investors updating their beliefs and bank creditors running will end after, at most, N iterations.

Stress-test simulations of the model are conducted by integrating the mechanisms within the Bank of Canada's MacroFinancial Risk Assessment Framework (MFRAF). This macro stress-testing tool kit is routinely used by the Bank of Canada to assess vulnerabilities within the Canadian banking sector. MFRAF, together with the information contagion channel, was used in the Bank of Canada's 2013 Financial Sector Assessment Program (FSAP) administered by the International Monetary Fund (see Anand et al., 2014). For this paper, we calibrate our model using balance-

sheet data for Canadian domestic systemically important banks. An extreme, but plausible stress scenario is used, which relates macroeconomic stresses across various economic sectors into credit and market losses on banks' loan and securities portfolios. We decompose the aggregate losses to the banking sector as those stemming from solvency risk, funding liquidity risk and information contagion. Numerical comparative static exercises are also conducted to demonstrate the sensitivity of the model to investors' beliefs and the price spread.

The rest of the paper is organized as follows. Some related literature is presented in Section 2. We present the basic model in Section 3. The feedback mechanism between funding and market liquidity is presented in Section 4. We illustrate the use of the mechanism for stress testing in Section 5. A final section concludes.

2 Related literature

Our paper bridges the gap between the theoretical literature on information contagion and the literature on stress-testing models.[2] Acharya and Yorulmazer (2008) consider a model with two banks, each with risk-averse depositors, where the returns on assets across the two banks have a common factor. The failure of one bank conveys adverse information about the common factor. This, in turn, increases the borrowing costs of the surviving bank. In order to maximize the joint probability of survival, both banks herd and undertake correlated investments. In contrast, we characterize the information contagion dynamics for a given asset structure of the banking sector and costs of funding.

[2]Contagion is generally divided into two types of propagation mechanisms: balance-sheet effects (see Kiyotaki and Moore, 2002 for an example) and information effects, whereby poor results at one institution lead creditors at other institutions to revise their prior beliefs.

Our paper is closely related to Chen (1999), who provides a model of contagious bank runs. Uninformed depositors of surviving banks conclude that informed depositors have withdrawn their funds from failed banks, which signals poor prospects for the banking industry. In our model, all depositors are imperfectly informed and contagion is a consequence of the feedback between the investors' beliefs about the quality of a bank's assets and the coordination friction between the bank's depositors.

In a more recent contribution, Li and Ma (2013) developed a similar model to ours, where mutually reinforcing bank runs and fire sales arise due to an adverse selection problem. The authors derive a joint equilibrium for the pooling price of assets and the failure conditions for banks. In contrast, we design an iterative model where the market prices and failure conditions are determined sequentially. This structure is more conducive to implementation within macro stress-testing models.

Notwithstanding their growing popularity, most macro stress-testing models do not capture the myriad of complex interactions between banks, and between banks and markets. Elsinger et al. (2006) propose one of the first models for stress testing systemic risk that quantifies contagion effects through interbank linkages and common assets. We omit interbank linkages for simplicity. Alessandri et al. (2009) provide a framework that incorporates credit risk and funding liquidity risk that materializes once banks' balance sheets deteriorate beyond certain exogenous thresholds. Gauthier et al. (2014) build on this framework and provide analytical underpinnings for the link between solvency risk and funding liquidity risk. Other models, such as Cifuentes et al. (2005) and Gauthier et al. (2012), introduce asset fire sales, where banks sell assets in a market with inelastic demand, resulting in falling prices and forcing other banks to sell assets as well.

3 Basic model

The model extends over three dates, $t = 0, 1, 2$ and is populated by $N \in \mathbb{N}$ leveraged financial institutions – hereafter called banks – and a large pool of risk-neutral creditors and investors. Each creditor is endowed with a unit of funding at $t = 0$ and is indifferent between consuming at $t = 1$ and $t = 2$. The representative investor, in contrast, is deep-pocketed and consumes only at $t = 2$.

As in Rochet and Vives (2004), at $t = 0$, bank i has E_i worth of internal financing, i.e., equity, and attracts additional debt from a mass D_i of creditors. Each creditor contributes a unit of funding, and in return receives a demandable debt contract that promises to repay $1 + r_i$ at the final date, where $r_i > 0$. The contract permits creditors to demand repayment at $t = 1$ without any penalty.[3]

With the funds $D_i + E_i$, bank i invests an amount M_i in liquid assets, such as treasury bills, and the remainder, $I_i \equiv D_i + E_i - M_i$, in risky assets, such as loans or securities. Risky assets mature at $t = 2$ and yield $Y_i > I_i$.[4] Table 1 depicts bank i's balance sheet at the end of $t = 0$.

	Assets	Liabilities	
Risky assets	I_i	D_i	Demandable debt
Liquid assets	M_i	E_i	Equity

Table 1: Balance sheet of bank i at $t = 0$

The risky assets of all banks are drawn from the same pool, which is either "high" quality or "low" quality. Assets from the high-quality pool (henceforth simply

[3] We take contracts as given and do not investigate their optimal design.

[4] To focus our attention on the ex post contagion dynamics, we abstract from the optimal portfolio structure in the paper.

referred to as high-quality assets) are worth $\psi_H \in (0, 1]$ to the investor, while low quality assets are worth only $\psi_L < \psi_H$. The quality of risky assets is determined at random at the start of $t = 1$. While banks observe the quality of assets, the investor and creditors do not. Instead, the investor has a common knowledge prior belief, $w \in [0, 1)$, that risky assets are high quality. The pooling price at which the investor offers to purchase assets at $t = 1$ is

$$\overline{\psi} = w\psi_H + (1 - w)\psi_L. \tag{1}$$

Banks suffer negative shocks to their balance sheets at $t = 1$ and $t = 2$. These shocks may be thought of as the materialization of credit risks on loans or market losses on trading assets. The shocks to bank i's assets at $t = 1$ and $t = 2$ are denoted by $S_i \in [\underline{S}_i, \overline{S}_i]$ and $L_i \in [\underline{L}_i, \overline{L}_i]$, respectively.[5] The $t = 1$ shock is distributed according to the probability distribution $f_i(S)$, with the cumulative distribution function given by $F_i(S)$. The $t = 2$ shock is distributed according to the probability distribution function $g_i(L)$, where the cumulative distribution function is denoted by $G_i(L)$.

Assets	Liabilities
$Y_i - S_i - L_i$	$D_i(1 + r_i)$
M_i	$\max\{E_i - S_i - L_i - D_i(1 + r_i), 0\}$

Table 2: Balance sheet of bank i at $t = 2$

Bank i is insolvent at $t = 2$ if its capital is insufficient to cover the losses; i.e., $S_i + L_i > E_i$. If $S_i > E_i$, then bank i is insolvent at $t = 1$. For $S_i \leq E_i$, bank i is solvent at $t = 1$ but can become illiquid if the fraction of creditors who demand

[5] In what follows, we assume that the support for the shock at $t = 2$ is entirely contained in the support for the shock at $t = 1$.

repayment i.e., "withdraw," is too large for the bank to service. A bank that is illiquid at $t = 1$ is assumed to be insolvent at $t = 2$. Table 2 depicts bank i's balance sheet at $t = 2$ when it is solvent.

Denoting the fraction of creditors who withdraw at $t = 1$ by $\ell_i \in [0, 1]$, bank i is illiquid whenever

$$\ell_i > \lambda_i\left(S_i\,;\,\overline{\psi}\right) \equiv \frac{M_i + \overline{\psi}\left[Y_i - S_i\right]}{D_i}. \tag{2}$$

The fraction $\lambda_i\left(S_i\,;\,\overline{\psi}\right)$ is the ratio of bank i's recourse to liquidity at $t = 1$ to its debts. The recourse to liquidity is the sum of the liquid assets and the proceeds from selling the remaining illiquid assets to the investor at the pooling price. Table 3 provides the timeline of events.

$t = 0$	$t = 1$	$t = 2$
1. Debt issuance	1. Interim shock	1. Investment matures
2. Investments	2. Debt withdrawals	2. Final shock
		3. Debts honored

Table 3: Timeline of events

3.1 Rollover decisions

We model the decisions of bank i's creditors to withdraw or not to withdraw their debt at $t = 1$ as a binary-action simultaneous move game. Table 4 provides the payoffs for an individual creditor.

If the creditor withdraws, then the bank returns the unit of funds, which is

	Solvent (at $t = 2$)	Insolvent (at $t = 2$) or Illiquid (at $t = 1$)
Not to withdraw	$1 + r_i$	0
Withdraw	1	1

Table 4: Payoffs for a creditor from rolling over its claims or withdrawing

immediately consumed. The creditor receives this payoff regardless of the outcome for the bank.[6] On the other hand, if the creditor decides to not withdraw, then the payoff depends on the outcome for the bank. If the bank is solvent at $t = 2$, then the bank repays the creditor the principal plus the interest r_i. However, if the bank is insolvent, or turns illiquid at $t = 1$, the creditor gets nothing.

Following the realization of S_i at $t = 1$, the bank is insolvent at $t = 2$ for realizations L_i that yield $E_i - S_i - L_i < 0$. Thus, conditional on the loss S_i, the probability that the bank will be insolvent at $t = 2$ is

$$\mathcal{N}_i(S_i) = \text{Prob}\big(E_i - S_i - L_i < 0\big) = \begin{cases} 1 & \text{if} \quad S_i > E_i - \underline{L}_i \\ 1 - G_i(E_i - S_i) & \text{if} \quad S_i \in \big[E_i - \overline{L}_i, E_i - \underline{L}_i\big] \\ 0 & \text{if} \quad S_i < E_i - \overline{L}_i \end{cases} . \tag{3}$$

If $S_i > E_i - \underline{L}_i$, then, for all realizations of the loss L_i, the bank is always insolvent at $t = 2$. Hence, it is a dominant strategy for all creditors to withdraw. On the other hand, if $S_i < E_i - \overline{L}_i$, the bank is always solvent at $t = 2$, and it is dominant for all creditors to not withdraw their claims against bank i. In the intermediate range, there is a non-zero probability of insolvency at $t = 2$ for bank i. With common knowledge over the loss S_i, the model exhibits multiple equilibria in pure strategies,

[6] This simplifying assumption allows us to preserve the global strategic complementarities assumption needed to solve the coordination game.

where either all creditors withdraw their claims, or no creditor withdraws. Figure 1 depicts this tripartite classification of the support for the shock S_i.

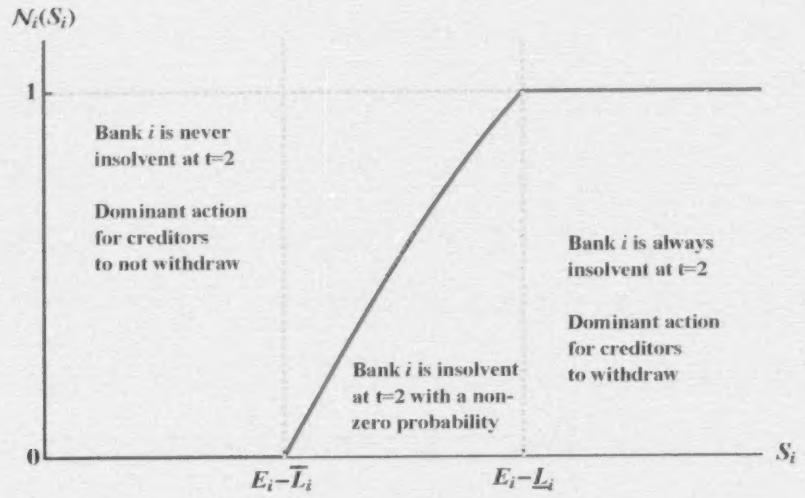
$N_i(S_i)$
1
0
Bank *i* is never insolvent at t=2
Dominant action for creditors to not withdraw
Bank *i* is insolvent at t=2 with a non-zero probability
Bank *i* is always insolvent at t=2
Dominant action for creditors to withdraw
S_i
$E_i - \overline{L}_i$
$E_i - \underline{L}_i$

Figure 1: Tripartite classification of the shock S_i for bank i.

3.2 Global games refinement

To resolve the multiplicity of equilibria, we employ the global games refinement (Morris and Shin, 2003), where the model is embedded in an incomplete information setting. Each creditor, $k \in [0, D_i]$, of bank i observes a noisy signal, $x_k = S_i + \epsilon_k$, on the realized losses, where ϵ_k is uniformly distributed over the interval $[-\epsilon, \epsilon]$.[7] We postulate that all creditors follow threshold strategies; i.e., there exists an x_i^*, such that if creditor k receives a signal $x_k > x_i^*$, then the creditor withdraws its claims against bank i. However, for $x_k < x_i^*$, the creditor does not withdraw. From the signal x_k, creditor k infers the distribution of other creditors' signals, and thus the likelihood that they have decided to withdraw. The following proposition summarizes.

[7]The support for the S_i shock distribution satisfies, $\underline{S}_i < E_i - \overline{L}_i - \epsilon$, and $\overline{S}_i > E_i - \underline{L}_i + \epsilon$.

Proposition 1. Critical illiquidity threshold. *In the limit of vanishing private noise, $\epsilon \to 0$, there exists a unique equilibrium in threshold strategies characterized by an illiquidity threshold, S_i^*, and a signal threshold, S_i^*, for each bank i. Creditor j withdraws the claim against bank i if and only if $x_j > x_i^*$, and bank i is illiquid if and only if $S_i^1 > S_i^*$, where $x_i^* \to S_i^*$. The illiquidity threshold is implicitly defined by the indifference condition for the expected payoff to a creditor between rolling over and withdrawing:*

$$G_i\big(E_i - S_i^*\big)\,\lambda_i\big(S_i^*\,;\overline{\psi}\big) = \frac{1}{1+r_i}. \tag{4}$$

The illiquidity threshold is increasing in the bank's equity, E_i, liquid assets, M_i, and returns, r_i, but is decreasing in its debt level, D_i. The threshold is increasing in the investor's belief, w, and the prices, ψ_H and ψ_L.

Proof. See Appendix A. □

The comparative statics for the critical thresholds are intuitive. First, as the bank's equity, E_i, increases, it has more resources to withstand the shocks on the interim and final dates. The bank's probability of turning insolvent at $t = 2$ is reduced, and hence, the creditors are more willing to not withdraw. Second, as the level of liquid assets, M_i, increases, the bank's recourse to liquidity is greater, and the bank is more able to meet withdrawals at the interim date. The creditors are more willing to not withdraw their debts. Third, as the promised return, r_i, increases, creditors are better off not withdrawing their debts and waiting until $t = 2$, than withdrawing at $t = 1$ and obtaining only unity. Fourth, as the debt level, D_i, increases, the bank has less recourse to liquidity to meet all withdrawals. This, in turn, prompts creditors to withdraw earlier.

Finally, increases in investors' belief, w, and prices, ψ_H and ψ_L, all lead to

increases in the pooling price, which in turn improves the bank's recourse to liquidity. As the bank has more liquidity to meet withdrawals, the creditors are willing to roll over their claims to the bank.

4 Contagion and self-fulfilling illiquidity

A bank's recourse to liquidity depends on the investor's belief about the quality of the bank's assets. If the investor is pessimistic to start with, this may precipitate some banks turning illiquid at $t = 1$. On observing these outcomes, the investor updates its belief and turns more pessimistic, which reduces other banks' recourse to liquidity. In a dynamic setting, this may lead to more banks turning illiquid, thereby making the investor more pessimistic.

This vicious cycle of self-fulfilling illiquidity between the investor updating the belief and the actions of bank creditors terminates once there is no additional information to be gained by the investor from observing the outcomes for banks. We explore this dynamic by first concentrating on two banks (i and j), where we divide $t = 1$ into two periods – round 1 and round 2.

Round 1: Investors have a prior belief $w^1 > 0$ that the banks' assets are of a high quality. The pooling price is $\overline{\psi}^1 = w^1\psi_H + \left(1 - w^1\right)\psi_L$. As in the basic set-up, creditors of banks i and j receive noisy signals on the shocks S_i and S_j, respectively. Formally, each creditor, $b_k \in D_b$, of bank $b \in \{i, j\}$ receives a signal, $x^1_{b_k} = S_b + \epsilon^1_{b_k}$. The noise term, $\epsilon^1_{b_k}$, is independently and identically drawn from the interval $[-\epsilon^1, \epsilon^1]$.

We focus on the global games solution in round 1, in the limit of vanishing private noise ($\epsilon^1 \to 0$). Following the logic of Proposition 1, there exist unique

illiquidity thresholds S_i^{1*} and S_j^{1*} for banks i and j, respectively. Bank $b \in \{i, j\}$ is illiquid at the end of round 1 if $S_b > S_b^{1*}$. There are four possible outcomes: (i) bank i is illiquid, but bank j is liquid; (ii) bank i is liquid, but bank j is illiquid; (iii) both bank i and bank j are illiquid; and (iv) both bank i and bank j are liquid.

Round 2: If both banks are illiquid at the end of round 1, then there is no further activity in round 2. Thus, without loss of generality, we focus on just two cases: (i) bank i is liquid, but bank j is illiquid; and (ii) both bank i and bank j are liquid. We introduce the indicator variable $\eta_b^1 \in \{0, 1\}$, which indicates whether bank b is liquid ($\eta_b^1 = 0$) or illiquid ($\eta_b^1 = 1$) at the end of round 1. On observing the outcomes, the investor updates its belief about the quality of the assets using Bayes' rule:

$$w^2 = \frac{\text{Prob}\Big(\eta_i^1\,,\,\eta_j^1 \,|\, \psi = \psi_H\Big)\, w^1}{\text{Prob}\Big(\eta_i^1\,,\,\eta_j^1 \,|\, \psi = \psi_H\Big)\, w^1 + \text{Prob}\Big(\eta_i^1\,,\,\eta_j^1 \,|\, \psi = \psi_L\Big)\,\Big(1 - w^1\Big)}\,. \tag{5}$$

Since the event η_i^1 is independent of the event η_j^1, the joint probability $\text{Prob}\Big(\eta_i^1\,,\,\eta_j^1 \,|\, \psi = \psi_H\Big) = \text{Prob}\Big(\eta_i^1 \,|\, \psi = \psi_H\Big)\,\text{Prob}\Big(\eta_j^1 \,|\, \psi = \psi_H\Big)$. The probability that bank $b \in \{i, j\}$ is illiquid ($\eta_b^1 = 1$), conditional on $\psi = \psi_H$, is

$$\text{Prob}\big(\eta_b^1 = 1 \,|\, \psi = \psi_H\big) = \text{Prob}\big(S_b > S_{bH}^*\big)\,, \tag{6}$$

where the critical threshold S_{bH}^* is given by the solution to

$$G_b\big(E_b - S_{bH}^*\big)\,\lambda_b\big(S_{bH}^*\,;\,\psi_H\big) = \frac{1}{1 + r_b}\,. \tag{7}$$

An analogous definition holds for S_{bL}^*. Once the new belief has been computed, a new pooling price, $\overline{\psi}^2 = w^2\,\psi_H + \big(1 - w^2\big)\,\psi_L$, is offered. Creditors of the remaining liquid bank(s) have the opportunity to revise their decision. They receive a new noisy signal

on the shock to their bank and decide whether to withdraw or not. Formally, creditor b_k of bank $b \in \{i, j\}$ receives a signal, $x^2_{b_k} = S_b + \epsilon^2_{b_k}$, where $\epsilon^2_{b_k}$ is independently and identically drawn from $[-\epsilon^2, \epsilon^2]$. The new noise term, $\epsilon^2_{b_k}$, is independent of the noise term from the previous round. We employ the global games solution in round 2 in the limit of vanishing private noise ($\epsilon^2 \to 0$). We thus define new illiquidity thresholds, S^{2*}_i and S^{2*}_j, for banks i and j, respectively. Whether or not the new thresholds are higher or lower than those from round 1 depends on how the investor's beliefs are updated. Table 5 provides the timeline for the game.

$t = 0$	$t = 1$ (round 1)	$t = 1$ (round 2)	$t = 2$
1. Debt issuance	1. Interim shock	1. Belief updated	1. Investment matures
2. Investments	2. Private signals	2. New pooling price	2. Final shock
	3. Debt withdrawals	3. New private signals	3. Debts honored
		4. Debt withdrawals	

Table 5: Timeline of events where the investor updates the belief

Proposition 2. Virtuous liquidity. *If both banks are liquid at the end of round 1, then $w^2 > w^1$. Consequently, both banks remain liquid at the end of round 2.*

Proof. See Appendix B. □

Suppose that at the start of round 1, the investor has a high prior belief that assets are good quality; i.e., w^1 is large. This, in turn, results in both banks remaining liquid at the end of round 1. The investor observes these outcomes at the start of round two and updates the belief upwards, since all banks are more likely to remain

liquid when assets are of good quality; i.e., $w^2 > w^1$. The banks' recourse to liquidity improves, which further increases the critical thresholds. The banks thus continue to remain liquid and solvent.

Proposition 3. Vicious illiquidity. *Suppose bank i is liquid and bank j is illiquid at the end of round 1. The investor become more pessimistic, $w^2 < w^1$, whenever:*

$$\frac{\text{Prob}\left(\eta_i^1 = 0 \,|\, \psi = \psi_H\right)}{\text{Prob}\left(\eta_i^1 = 0 \,|\, \psi = \psi_L\right)} < \frac{\text{Prob}\left(\eta_j^1 = 1 \,|\, \psi = \psi_L\right)}{\text{Prob}\left(\eta_j^1 = 1 \,|\, \psi = \psi_H\right)}. \tag{8}$$

If the downward revision of the belief is large enough, then bank i will also become illiquid at the end of round 2.

Proof. See Appendix C. □

Suppose the investor enters round 1 with a pessimistic view regarding the quality of the banks' assets; i.e., w^1 is low, which results in bank j turning illiquid. When updating its belief at the start of round 2, the investor computes and compares four different conditional probabilities. If the inequality in Equation (8) is satisfied, the investor concludes that it is less likely for bank i to remain liquid when assets are good quality than it is for bank j to turn illiquid when assets are bad quality. The failure of bank j has a greater weight in the investor's Bayesian update, leading to a downward revision of the belief and a more pessimistic view on asset quality.

To investigate the effects of changes in the prices of good- and bad-quality assets, we make the following assumptions. Both banks are identical in all aspects, except in their reliance on short-term funding; i.e., $D_j > D_i$. The shock distribution $f = f_i = f_j$ satisfies $f' < 0$, such that small shocks are more likely than large shocks.

Finally, the ex ante probability that bank j will turn illiquid when assets are high quality is bounded above, $1 - F(S^*_{jH}) < \bar{F}$, where $\bar{F}$ is defined in Appendix D.

Proposition 4. Price and spread effects. *For a given initial belief, w^1, and low-quality price, ψ_L, an increase in the high-quality price, ψ_H, increases the price spread, $\Delta\psi = \psi_H - \psi_L$. This, in turn, strengthens the pessimism condition and increases the range of parameters where the investor's belief is revised downwards.*

On the other hand, for a given high-quality price, ψ_H, an increase in the low-quality price, ψ_L, leads to a decrease in the price spread. This weakens the pessimism condition and reduces the range of parameters where the investor's belief is revised downwards.

Proof. See Appendix D. □

As the price spread increases, the probability that bank i remains liquid when assets are good quality, relative to the probability that bank i remains liquid when the assets are bad quality, also increases. At the same time, the probability that bank j turns illiquid when assets are bad quality, relative to the probability that bank j turns illiquid when assets are good quality, also increases. When the increase in the relative probability that bank j turns illiquid when assets are bad quality is greater than the increase in the relative probability that bank i remains liquid when assets are good quality, the net effect is to increase the range of parameters where the pessimism condition holds. The opposite effect holds when the price spread decreases.

With the updated beliefs, there are two possible outcomes at the end of round 2: bank i remains liquid or turns illiquid. If bank i remains liquid, there is no additional information for investors to gather, and their beliefs remain unchanged. On the

other hand, if bank i turns illiquid, investors will become more pessimistic. In both cases, there are no further actions possible in subsequent rounds. Generalizing to an arbitrary number, N, of banks, the following proposition summarizes the result.

Proposition 5. Convergence. *In a game involving $N \geq 2$ banks, the cycles of Bayesian updating by investors and withdrawal by creditors terminates after, at most, N rounds.*

Proof. See Appendix E. □

The result of Proposition 5 ensures that any algorithm that implements our model will be well behaved and converge after a finite number of rounds. This is vital for implementing our model for stress-testing purposes, which we explore in the next section.

5 Stress testing and simulations

Our analysis thus far provides a theoretical underpinning for the links between solvency risk, funding liquidity risk and contagion. We now illustrate these links by integrating our model into the Bank of Canada stress-testing model: the MacroFinancial Risk Assessment Framework (MFRAF). Figure 2 provides a high-level overview of MFRAF. This integrated framework has recently been implemented by the Bank of Canada and is used to assess the resilience of the Canadian banking sector.[8]

[8] In particular, results from the 2013 Financial Sector Assessment Program, an exercise administered by the International Monetary Fund, have been published by Anand et al. (2014). We provide only a very brief discussion here – for further details, see Gauthier et al. (2012) and Gauthier et al. (2014).

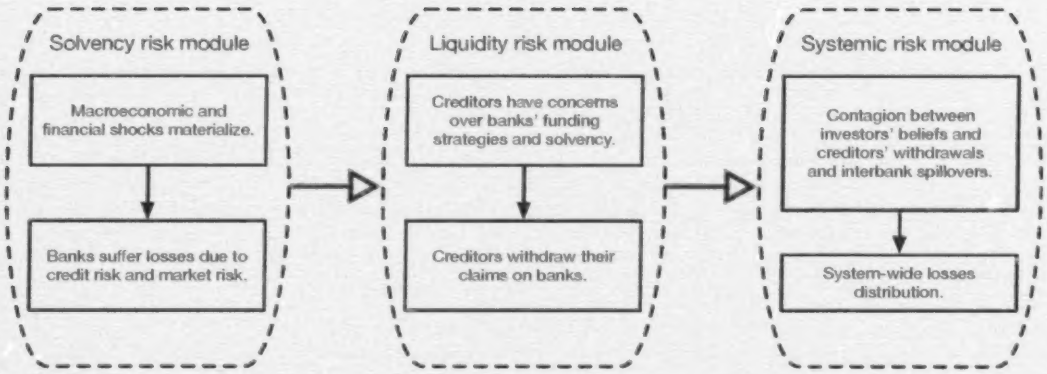
Solvency risk module
Macroeconomic and financial shocks materialize.
Banks suffer losses due to credit risk and market risk.
Liquidity risk module
Creditors have concerns over banks' funding strategies and solvency.
Creditors withdraw their claims on banks.
Systemic risk module
Contagion between investors' beliefs and creditors' withdrawals and interbank spillovers.
System-wide losses distribution.

Figure 2: Schematic description of the modular structure of MFRAF

The framework involves three independent but interdependent modules. First, banks are subjected to shocks, leading to asset losses over a one-year horizon. These shocks, which can originate in the real economy (adverse macroeconomic shocks) or in financial markets, weaken banks' capital positions and threaten their future solvency. Banks can be affected differently, depending on their initial balance-sheet vulnerabilities (asset quality, leverage, funding strategies, etc.). Second, as initial losses materialize at the interim date, concerns about their future solvency mount, causing creditors to withdraw their loans, thereby generating funding liquidity risk. In the third step, the illiquidity of one bank can lead to other banks' illiquidity due to information contagion through investors' beliefs.[9]

5.1 Calibration

We illustrate MFRAF in a banking system consisting of six banks that represent the Canadian domestic systemically important banks (D-SIBs). The macroeconomic

[9]MFRAF also incorporates network externalities caused by defaults by counterparties. A defaulting bank (or a bank with a serious capital shortfall) will not be able to fulfill its obligations in the interbank market, causing counterparty credit losses in the system and leading to the potential default of other banks. These effects amplify the impact of initial shocks, but in this paper, we ignore these externalities and instead focus on the information contagion channel.

stress scenario loosely draws on the baseline scenario considered in Canada's 2013 FSAP (see Anand et al. (2014) and IMF (2014) for details). Our calibration uses confidential data on banks' balance sheets. For this reason, we only present aggregate results of the calibration that have been publicly reported.

The scenario considered a peak-to-trough real GDP decline of -5.9 percent. The peak increase in the unemployment rate was also 5.9 percent, and house prices declined by 33 percent. These macroeconomic trends were mapped into default rates for loans on banks' balance sheets. Different rates were derived for different economic sectors.[10] The average default rate across all sectors was 6.2 percent. Over the period 1988–2012, the historical average default rate was only 4.4 percent.

The stress horizon in our simulation is one year. Banks' annual losses were determined as follows. We simulate default rate distributions for different sectors using two components. The first is the average default rate for each sectors, which is given by our macroeconomic mapping. The second component is the correlation matrix of defaults across different sectors, which is derived from historical data. Next, information on banks' exposures to the different sectors is obtained from regulatory filings. Finally, data on loss-given-default rates are obtained from banks. IMF (2014) reports that the peak of the weighted-average loss-given-default rate over the stress horizon was around 33 percent. Annual losses for each bank is the product of the default rates, exposures to the various sectors and the loss-given-default rates. In our stress test, we assume that one-half of the annual losses are realized at the interim date. The remainder are realized at the final date.

Banks' balance sheets are calibrated using data from the first quarter of 2013.

[10]The domestic sectors include accommodations, agriculture, construction, manufacturing, wholesale, Canadian governments, financial institutions, small business loans, residential mortgages (uninsured), HELOCs (uninsured) and consumer loans.

The average common equity Tier 1 (CET1) capital ratio of banks is 8.9 percent. Additional data on the liquid assets, illiquid assets and risk-weighted assets are obtained from banks' regulatory filings. To estimate the amount of funding that is subject to rollover risk at the interim date in our model, we consider both short- and long-term liabilities, regardless of their original contract maturities. We obtain data on banks' maturity mismatches from the Office of the Superintendent of Financial Institutions. We use a subset of this data, which covers debts that are coming to maturity within a six-month horizon. All these liabilities are subject to withdrawal at the interim date in our model. On average, these liabilities represent 35 percent of total liabilities across all six banks. The returns on banks' liabilities are calculated as weighted averages of the returns across different instruments and maturities. The values of good- and bad-quality assets to investors in the model are equal to the weighted averages of the assets' liquidation values during times of stress. We draw on the views of market experts to determine the liquidation values. We thus set $\psi_H = 0.3$ and $\psi_L = 0.2$.

The system-wide loss distribution is determined as follows. Insolvent banks' losses are equal to their credit shocks plus a bankruptcy cost, which is equal to 10 percent of their risk-weighted assets. Banks that fail due to illiquidity suffer an additional bankruptcy cost equal to the spread between ψ_H and the average pooling price offered by investors, multiplied by the bank's illiquid assets. The losses are proportional to the level of adverse selection.

To concentrate on how balance-sheet characteristics influence liquidity risk, we make the following simplifying assumptions. The loss distributions are identical for all banks and equal to the loss distribution for the average bank. All banks have the same CET1 capital, liquid assets, illiquid assets and promised returns. Banks, however, differ in terms of the liabilities subject to withdrawal at the interim date.

5.2 Stress-test results

The results of our stress test are provided in Table 6. The probability that the capital ratios of banks would breach the minimum regulatory requirement of 7 percent following a credit shock is 47 percent for all banks. In crisis times, uncertainty about banks' assets increases, and creditors become highly sensitive to breaches of the minimum capital requirements. Therefore, a bank that breaches the minimum regulatory requirement is assumed to be insolvent from the perspective of creditors.[11]

	Risks			
Bank	Solvency	Liquidity	Contagion	Total
1	47.0	22.9	0.0	69.9
2	47.0	0.0	0.0	47.0
3	47.0	23.0	0.6	70.6
4	47.0	0.0	19.2	66.2
5	47.0	0.0	0.0	47.0
6	47.0	22.2	0.8	70.0

Table 6: Decomposition of risks

A bank's *balance sheet liquidity* (BSL) is the ratio of its recourse to liquidity – the sum of liquid assets and the sale of illiquid assets at the initial pooling price – to it liabilities subject to rollover risk.[12] Assuming that the investor holds a prior belief,

[11] Of course, using a standard criterion of negative net worth would yield lower solvency and liquidity risks.

[12] The BSL measure is different from the LCR and should not be viewed as a proxy for the LCR. The denominator for the BSL represents liabilities subject to withdrawal at the interim date in the model and is calculated as all liabilities with a maturity date falling within six months of the start of the exercise. The LCR, on the other hand, considers cash outflows over a one-month horizon only. The two measures also differ in their assumptions on the proportion of liabilities that are subject to withdrawal, and on the haircuts to illiquid assets.

$w^1 = 1/2$, the average BSL is $\overline{\lambda} = 1.08$, with three banks (banks 2, 4 and 5) having a BSL ratio larger than one. For moderate shocks, these banks never suffer illiquidity in round 1, whereas banks 1, 3 and 6 have non-zero probabilities of turning illiquid in round 1.

We also compute for each bank the probability of turning illiquid when the investors revise their beliefs. At the end of round 1, the investors observe the failure of banks 1, 3 and 6 and turn very pessimistic. This, in turn, reduces bank 4's recourse to liquidity, which reduces its BSL ratio to less than one. Consequently, the probability that bank 4 will turn illiquid after round 1 is non-zero and equal to 19.2 percent.

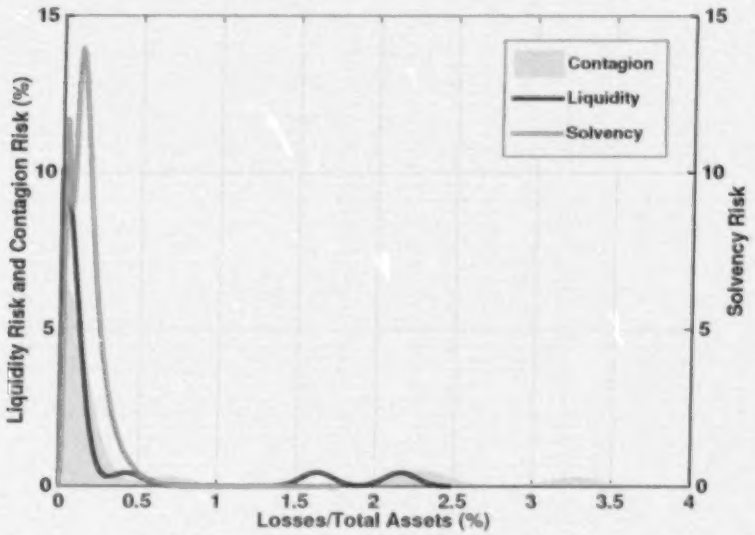

Figure 3: Aggregate loss distribution

Figure 3 shows the impact of the three types of risk on the distribution of the aggregated losses as a percentage of total assets for the banking system. When only the direct impact of the initial shocks is considered (the red line, labeled "Solvency"),

maximum system-wide losses do not exceed 2 percent of total assets, and average losses amount to less than 0.5 percent of total assets. The tail is, however, significantly fattened by adding funding liquidity risk to credit risk (blue line, labeled "Liquidity"). The likelihood of the banking system suffering losses larger than 2 percent of its total assets is non-zero. Including contagion effects (the light-blue shaded area, labeled "Contagion") worsens the system loss distribution (losses can now reach up to 6 percent of the system's assets). These results demonstrate that the failure to account for contagion risk could significantly underestimate the extent of systemic risk: banks considered liquid when analyzed in isolation may become illiquid due to information contagion.

Table 7 presents the results for a second set of simulations where we increase the stock of liabilities subject to withdrawal for banks 2 and 5, such that their BSL ratios are equal to that of bank 4. Banks 2 and 5 continue to remain immune to liquidity risk in round 1. However, following the investors' revisions, the pooling price falls and their BSL ratios fall below 1. This, in turn, increases the probability of both banks suffering from illiquidity due to contagion.

5.3 Beliefs and prices in stress testing

As a final exercise, we investigate how changes in the prior belief, w^1 and the price spread, $\psi_H - \psi_L$, influence liquidity risk and contagion in our stress-testing exercise. The top panel of Figure 4 plots the total liquidity risk probability as a function of the investors' initial beliefs. As the investors become more optimistic that assets are of high quality, the liquidity risk probability diminishes. The three different curves in the plot represent changes to the low-quality price, ψ_L. For the 10 percent (20

	Risks			
Bank	Solvency	Liquidity	Contagion	Total
1	47.0	22.9	0.0	69.9
2	47.0	0.0	22.6	69.6
3	47.0	23.0	0.6	70.6
4	47.0	0.0	19.2	66.2
5	47.0	0.0	19.7	46.7
6	47.0	22.2	0.8	70.0

Table 7: Decomposition of risks with lower BSLs for banks 2 and 5

percent) curve, the price ψ_L is reduced by 10 percent (20 percent) relative to the baseline case, which is represented by the 0 percent curve. A decrease in the low-quality price leads to an increase in the price spread. As the figure demonstrates, liquidity risk is higher when the price spread is high as well.

The middle panel of Figure 4 depicts the liquidity risk after the first round of the information contagion dynamics. As the prior belief increases, the first-round liquidity risk decreases more rapidly than the total risk. However, at the same time, as the bottom panel of Figure 4 demonstrates, this decline in the first-round liquidity risk is supplemented by an increase in the contagion risk. When the price spread is large, the first-round liquidity risk is higher for larger values of the belief. As the belief continues to increase, this liquidity risk is replaced by contagion risk, to keep the total liquidity risk figure unchanged. This numerical finding is consistent with the result in Proposition 4.

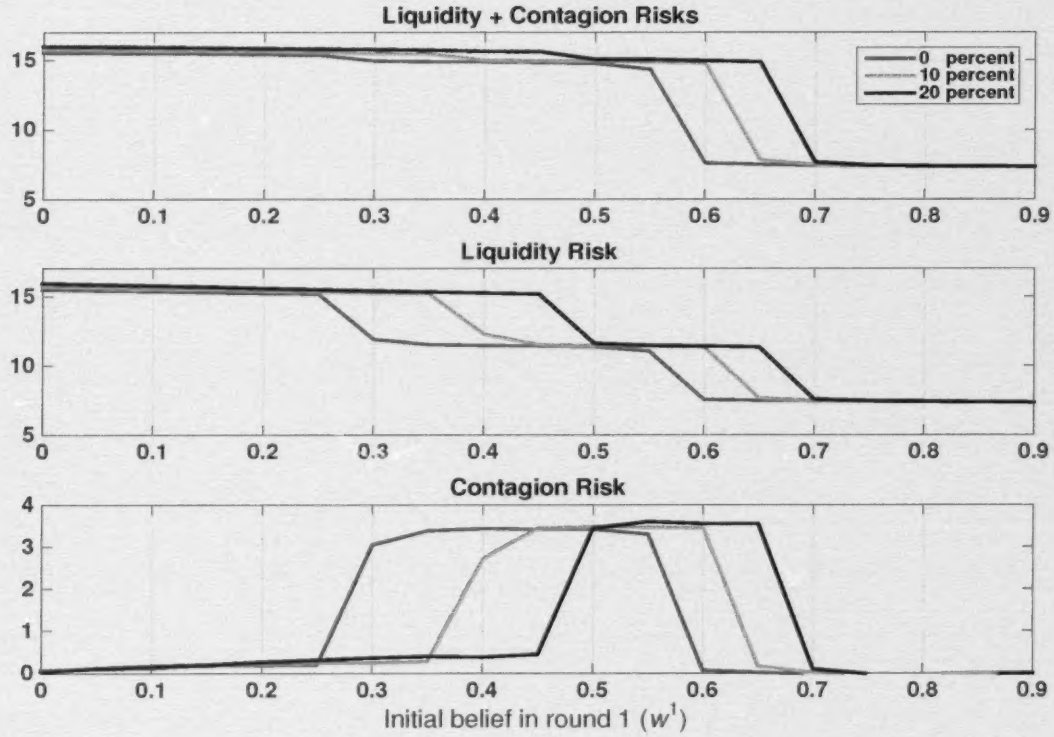

Figure 4: Decomposition of liquidity risk and contagion

6 Conclusion

In this paper, we propose a tractable model-based stress-testing framework that integrates solvency risk, funding liquidity risk and market risk. The key dynamic underlying our model is a two-way feedback interaction between the adverse selection of investors seeking to purchase assets and coordination failure between the creditors of banks that leads to endogenous runs. The investors' initial belief is crucial for the unfolding of systemic risk. In particular, if investors are pessimistic, this reduces banks' recourse to liquidity – from selling illiquid assets – which makes them more susceptible to runs. This, in turn, makes investors more pessimistic. We illustrate these dynamics in a stress-testing exercise using an extreme but plausible macroeconomic

stress scenario.

The global financial crisis highlighted the mutually reinforcing roles of funding and market liquidity when banks' balance sheets lack transparency. Tools such as ours that can be used to better assess the interactions between different risks in the financial system can help limit the recurrence of such events.

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A Proof of Proposition 1

We establish the existence of a unique Bayesian equilibrium for each bank, b, in threshold strategies. Morris and Shin (2003) show that only threshold strategies survive the iterated deletion of strictly dominated strategies.

Each creditor, b_k, uses a threshold strategy, whereby debt is rolled over if and only if the private signal suggests that the credit shock is small, $x_{b_k} < x_b^*$. Hence, for a given realization of the shock S, the proportion of creditors who do not roll over debt is

$$\ell_b(S_b, x_b^*) = \text{Prob}\big(x_{b_k} > x_b^* | S\big) = \text{Prob}\big(\epsilon_{b_k} > x_b^* - S_b\big) = 1 - \frac{x_b^* - S_b - \epsilon}{2\epsilon}. \quad (9)$$

The critical mass condition states that bank b is illiquid when the credit shock reaches a threshold S_b^*, such that

$$\ell_b(S_b^*, x_b^*) = \frac{M_b + \overline{\psi}\big[Y_b - S_b^*\big]}{D_b}. \quad (10)$$

A creditor who receives the signal x^* is indifferent between rolling over its debt and not. Thus, the indifferent condition is

$$G_b\big(E_b - S_b^*\big)\, \lambda_b\big(S_b^*\,;\, \overline{\psi}\big)\, \big(1 + r_b\big) = 1. \quad (11)$$

Equation (10) defines the critical signal, while Equation (11) defines the critical threshold for bank b. In the limit $\epsilon \to 0$, it is easy to verify that $x_b^* \to S_b^*$.

B Proof of Proposition 2

The belief at the end of round 1 is

$$\begin{aligned} w^2 \;=\; & \Big(\text{Prob}\big(\eta_i = 0 \,|\, \psi = \psi_H\big)\,\text{Prob}\big(\eta_j = 0 \,|\, \psi = \psi_H\big)\,w^1\Big) \\ \Big/ \; & \Big(\text{Prob}\big(\eta_i = 0 \,|\, \psi = \psi_H\big)\,\text{Prob}\big(\eta_j = 0 \,|\, \psi = \psi_H\big)\,w^1 \\ + \; & \text{Prob}\big(\eta_i = 0 \,|\, \psi = \psi_L\big)\,\text{Prob}\big(\eta_j = 0 \,|\, \psi = \psi_L\big)\,\big(1 - w^1\big)\Big). \end{aligned} \tag{12}$$

To show that $w^2 > w^1$, we must have that

$$\begin{aligned} & \text{Prob}\big(\eta_i = 0 \,|\, \psi = \psi_H\big)\,\text{Prob}\big(\eta_j = 0 \,|\, \psi = \psi_H\big) \\ \geq \; & \text{Prob}\big(\eta_i = 0 \,|\, \psi = \psi_H\big)\,\text{Prob}\big(\eta_j = 0 \,|\, \psi = \psi_H\big)\,w^1 \\ + \; & \text{Prob}\big(\eta_i = 0 \,|\, \psi = \psi_L\big)\,\text{Prob}\big(\eta_j = 0 \,|\, \psi = \psi_L\big)\,\big(1 - w^1\big), \end{aligned} \tag{13}$$

which, on rearranging, yields

$$\begin{aligned} & \text{Prob}\big(\eta_i = 0 \,|\, \psi = \psi_H\big)\,\text{Prob}\big(\eta_j = 0 \,|\, \psi = \psi_H\big) \\ > \; & \text{Prob}\big(\eta_i = 0 \,|\, \psi = \psi_L\big)\,\text{Prob}\big(\eta_j = 0 \,|\, \psi = \psi_L\big), \end{aligned} \tag{14}$$

which is always true.

C Proof of Proposition 3

The belief at the end of round 1 is

$$\begin{aligned} w^2 &= \Big(\text{Prob}\big(\eta_i = 0 \,|\, \psi = \psi_H\big)\,\text{Prob}\big(\eta_j = 1 \,|\, \psi = \psi_H\big)\,w^1\Big) \\ &/ \Big(\text{Prob}\big(\eta_i = 0 \,|\, \psi = \psi_H\big)\,\text{Prob}\big(\eta_j = 1 \,|\, \psi = \psi_H\big)\,w^1 \\ &+ \text{Prob}\big(\eta_i = 0 \,|\, \psi = \psi_L\big)\,\text{Prob}\big(\eta_j = 1 \,|\, \psi = \psi_L\big)\,\big(1 - w^1\big)\Big). \end{aligned} \tag{15}$$

Consequently, for $w^2 < w^1$, we must have that

$$\begin{aligned} &\text{Prob}\big(\eta_i = 0 \,|\, \psi = \psi_H\big)\,\text{Prob}\big(\eta_j = 1 \,|\, \psi = \psi_H\big) && (16) \\ <\ &\text{Prob}\big(\eta_i = 0 \,|\, \psi = \psi_L\big)\,\text{Prob}\big(\eta_j = 1 \,|\, \psi = \psi_L\big), && (17) \end{aligned}$$

which is identical to the condition in Equation (8).

D Proof of Proposition 4

Let us define

$$\rho_0 = \frac{F(p^*_{i,H})}{F(p^*_{i,L})}, \tag{18}$$

which is the left-hand side of the pessimism condition, and

$$\rho_1 = \frac{1 - F(p^*_{j,L})}{1 - F(p^*_{j,H})}, \tag{19}$$

which is the right-hand side. The derivative of ρ_0 with respect to ψ_H is

$$\frac{d\rho_0}{d\psi_H} = \frac{f(p^*_{i,H})}{F(p^*_{i,L})}\,\frac{dp^*_{i,H}}{d\psi_H} > 0, \tag{20}$$

while the derivative of ρ_1 with respect to ψ_H is

$$\frac{d\rho_1}{d\psi_H} = \frac{1 - F(p^*_{j,L})}{\Big(1 - F(p^*_{j,H})\Big)^2} f_j(p^*_{j,H}) \frac{dp^*_{j,H}}{d\psi_H} > 0\,. \tag{21}$$

If follows from the implicit function theorem that $\frac{dp^*_{j,L}}{d\psi_H} > \frac{dp^*_{j,H}}{d\psi_H}$. Moreover, since $f' < 0$, it follows that $f(p^*_{j,H}) > f(p^*_{i,H})$. Finally, we have that $\frac{d\rho_1}{d\psi_H} > \frac{d\rho_0}{d\psi_H}$ as long as

$$\frac{1 - F(p^*_{j,L})}{\Big(1 - F(p^*_{j,H})\Big)^2} > \frac{1}{F(p^*_{i,L})}\,. \tag{22}$$

Turning to the effects of a change in ψ_L:

$$\frac{d\rho_0}{d\psi_L} = -\frac{F(p^*_{i,H})}{F(p^*_{i,L})^2} f(p^*_{i,L}) \frac{dp^*_{i,L}}{d\psi_L} < 0\,, \tag{23}$$

and

$$\frac{d\rho_1}{d\psi_L} = \frac{-1}{1 - F(p^*_{j,H})} f(p^*_{j,L}) \frac{dp^*_{j,L}}{d\psi_L} < 0\,. \tag{24}$$

Comparing the two derivatives, as before, we have that $f(p^*_{j,L}) > f(p^*_{i,L})$, and that $\frac{dp^*_{j,L}}{d\psi_L} > \frac{dp^*_{i,L}}{d\psi_L}$. We thus obtain $\frac{d\rho_1}{d\psi_L} < \frac{d\rho_0}{d\psi_L}$ whenever

$$\frac{1}{1 - F(p^*_{j,H})} > \frac{F(p^*_{i,H})}{F(p^*_{i,L})^2}\,. \tag{25}$$

Combining Equations (22) and (25), we obtain the sufficient condition for our result that

$$1 - F(p^*_{j,H}) < \bar{F} \equiv \min\left\{ \frac{F(p^*_{i,L})^2}{F(p^*_{i,H})}, \sqrt{F(p^*_{i,L})\Big(1 - F(p^*_{j,L})\Big)} \right\}. \tag{26}$$

E Proof of Proposition 5

Base case: In the case $N = 2$ at the end of round 1, either both banks have turned illiquid, only one bank has turned illiquid or both remain liquid. In the first case, investors update their beliefs and become pessimistic, but there are no further actions to take. In the third case, Proposition 2 implies that investors become optimistic when they update their beliefs, and no banks suffer from illiquidity. Finally, if only one bank defaults, then investors may become more pessimistic when they update their beliefs. In the worst case, this will lead to the second bank turning illiquid in round 2, after which there are no further actions, and the game terminates.

Induction Hypothesis: In the case of $N > 2$ banks, the game terminates after, at most, N rounds.

Inductive Step: In the case of $N + 1$ banks, suppose that at the end of N rounds, there are $N + 1 - k$ banks liquid and k banks illiquid, where $k \leq N + 1$. If $k = N + 1$, then all banks are illiquid, and the game ends. If $k = N$, then for the lone liquid bank, in round $N + 1$, investors update their beliefs and post a new pooling price. The creditors of the bank subsequently decide whether or not to withdraw. If they do not withdraw, then the bank remains liquid, and there is no further information to be gained for the investors, and the game terminates. If, however, they all withdraw, then the bank turns illiquid. While investors update their beliefs, there are no further actions to take and, hence, the game also terminates. For $k < N$, it follows that in round $N + 1 - k$ there were no new banks turning illiquid, and, hence, beliefs did not update, implying that the game terminated.